Chromatography - “color” and “to write”

Originally described by Tswett - 1906
He devised a method to separate plant pigments using a tube filled with CaCO₃.
After adding a plant extract, he was able to produce several colored bands by washing the extract through the column with an organic solvent.

The separation process
Sample components are carried by a mobile phase through a bed of stationary phase.
Individual species are retarded by the stationary phase based on various interactions such as:
- Surface adsorption
- Relative solubility
- Charge

Frontal analysis
Continuously add your sample to the start of the column.
Monitor components as they evolve.
Gives a general measure of how things are retained.
Example - charcoal filtration

Frontal analysis
Approach can be used to evaluate relative retention. Not useful as a method of separation.

Displacement Analysis
Materials move down a column by being displaced by a more highly retained solute.
Example - ion exchange / water softeners

You can't achieve complete resolution. Making the column longer has no effect.
A solute partitions between two phases (equilibrium). Separation is based on relative retention. Making the column longer will increase the degree of separation.

In plate theory, we treat our chromatographic column as though it was a ‘static’ system in equilibrium. Each species exhibits an equilibrium between the mobile and stationary phase.

We can define a partition coefficient based on this equilibrium.

$$K = \frac{\text{concentration solute in stationary phase}}{\text{concentration solute in mobile phase}}$$

K is assumed to be independent of concentration. It can be altered by such factors as temperature. As K increases, our solutes take longer to pass through the column.

For a separation, assume that:

The column is of fixed length. Flow is held constant.

We can then describe several terms based on how our average sample species performs. The first thing to do is measure how long it takes for our solute to travel through our column.
Retention time and volume

Retention volume, \( V_R \) - volume of mobile phase required to elute a solute to a maximum from a column.

Retention time, \( t_R \), time required to reach the same maximum at constant flow.

\[ V_R \text{ or } t_R \]

where \( V_R = t_R \times \text{flowrate} \)

Plate theory

The average linear rate for the solute is:

\[ v = \frac{L}{t_R} \]

where \( L \) is the column length.

And for the mobile phase it is:

\[ u = \frac{L}{t_R} \]

This is simply the time required for a non-interacting substance to pass through the column.

Plate theory

\[ v = u \times f \]

\( f \) is the fraction of time that the solute spends in the mobile phase.

\[ f = \frac{n_{\text{solute}_M}}{n_{\text{solute}_S}} \]

Since both the mobile and stationary phases have known volumes \( (V_M, V_S) \), we can now determine \( K \) - our partition coefficient.

\[ v = u \left( \frac{1}{1 + \frac{K V_S}{V_M}} \right) \]

This shows the factors required to have a component elute and how two materials can be separated.

Each material has its own \( K \). As \( K \) increases, elution takes longer.

Other terms affect the overall separation.

Plate theory

\[ v = u \left( \frac{C_M V_M}{C_M V_M + C_M V_S} \right) \]

\[ = u \left( \frac{1}{1 + \frac{V_S}{V_M}} \right) \]

\[ = u \left( \frac{1}{1 + \frac{K V_S}{V_M}} \right) \]

Raising

\( V_S \) - General increase in retention
\( V_M \) - General decrease in retention
\( u \) - Increases speed of separation.

\( V_S \) and \( V_M \) can be altered by changing column diameter and length for a specific column packing.

\( u \) can be altered by changing the flowrate.

All terms can be found by knowing how the column was prepared.
We'd like to be able to simply inject a sample and obtain the information we need from the results.

**Capacity factor**

\[ k' = \frac{K V_s}{V_M} \] (constant for our column conditions)

so

\[ \nu = u \left( \frac{1}{1 + k'} \right) \]

We can now expand the equation.

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We now have a simple method to determine \( k \) values based on elution times.

\[ \frac{t_R}{t_M} = \frac{1}{1 + k'} \]

\[ k' = \frac{t_R - t_M}{t_M} \]

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Remember, the goal of chromatography is to ‘separate’ two or more species.

Since each species has its own \( k \) value, we need a way to tell if two species can be separated.

**Selectivity factor**

\[ K_b > K_a, \quad \alpha = \frac{K_B}{K_A}, \quad \alpha \geq 1 \]

We can also use:

\[ \alpha = \frac{k'_a}{k'_b} = \frac{t_{RB} - t_M}{t_{RA} - t_M} \]

You can either:

- Determine \( \alpha \) based on the retention times for your solutes or
- Estimate if your species are separable based on \( \alpha \).

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Mobile phase volume is proportional to column length so retention is also increased for longer columns.

However, as peaks travel through the column, they broaden. Width increases with the square root of column length.

You can't just make a column longer to obtain a ‘better’ separation.

**Theoretical plates - \( N \)**

- In solvent extraction, a plate is represented by each equilibrium (extraction) we conduct.
- In a chromatographic column, we can’t see these plates so they are theoretical.
- We can estimate the number of theoretical plates in our column based on peak retention times and widths.
- Both factors are important in determining if a separation will work.
The number of plates can be determined from the retention time and peak width.

\[ N = 16 \left( \frac{t_R}{W} \right)^2 \]

This approach is taken because peaks evolve as Gaussian-like shapes and can be treated statistically.

In essence, we are taking \( \pm 2 \sigma \).

In this example, we have materials with the same elution time but different numbers of plates.

\[ N = 1000 \]
\[ N = 100 \]

It is often difficult to accurately measure peak width.

- The peak may co-elute with another.
- Low detector sensitivity can result in you finding the start or end at the wrong place.
- We can take an alternate approach.

We can measure the width at half height.

This insures that we are well above background and away from any detector sensitivity limit problems.

Since the peak is Gaussian in nature, we end up with the following modified formula.

\[ N = 5.54 \left( \frac{t_R}{W_{1/2}} \right)^2 \]

For a fixed length column, we can calculate an additional term - \( h \) (or HETP)

\[ h = \text{height equivalent of a theoretical plate} = \text{column length} / N \]
Resolution

Knowing how well a column can retain a component is nice but we need to deal with multiple eluents or why bother.

**Resolution,** $R_s$

A measure of how completely two neighboring peaks are separated from each other.

Since $v = V_m (1 - k)$

resolution in terms of $k$ is:

$$R_s = \frac{N}{2} \left( \frac{k_2 - k_1}{2 + k_2 + k_1} \right)$$

Remember that the relative volatility, $\alpha$, is

$$\alpha = \frac{k_2}{k_1}$$

The exact resolution equation is:

$$R_s = \frac{N}{4} \ln \left( 1 + \frac{k_2 - k_1}{1 + k_1} \right)$$

$k_2 \approx k_1$, when $\frac{k_2 - k_1}{1 + k_1}$ is small compared to 1.

and $\ln (1 + x) \approx x$

For a small $x$, $R_s = \frac{N}{4} \left( \frac{k_2 - k_1}{1 + k_1} \right)$

Approximate resolution equations

Since $\alpha = k_2/k_1$, we get

$$R_s = \frac{N}{4} \left( \frac{\alpha - 1}{\alpha + 1 + 2/k_1} \right)$$

**Knox equation**

Similarly, if we start with

$$R_s = -\frac{N}{4} \ln \left( 1 - \frac{k_2 - k_1}{1 + k_1} \right)$$

This will lead to the **Purnell Equation**

$$R_s = \frac{N}{4} \frac{\alpha + 1}{\alpha} \frac{k_2}{1 + k_2}$$
Each version of the equation will yield $R_s$ values that deviate from the exact solution.

Knox - $R_s$ values will be higher
Purnell - $R_s$ values will be lower
( a more conservative estimate )

If we assume that the peaks are Gaussian, we can use:

$$R_s = 1.18 \frac{t_{R2} - t_{R1}}{W_{1/22} + W_{1/21}}$$

A 'working' form of the resolution equation.

### Determine the $k$, $N$ and HETP for toluene in the following analysis

<table>
<thead>
<tr>
<th>Solute</th>
<th>$t_R$, min</th>
<th>$W_{1/2}$, min</th>
</tr>
</thead>
<tbody>
<tr>
<td>air</td>
<td>1.5</td>
<td></td>
</tr>
<tr>
<td>benzene</td>
<td>7.45</td>
<td>1.05</td>
</tr>
<tr>
<td>toluene</td>
<td>10.6</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Column length = 10 meters
Flow rate = 30 ml/min
Isothermal conditions

$$k = \frac{t_R}{t_M} - 1$$

$$N = 5.54 \left( \frac{t_R}{W_{1/2}} \right)^2$$

$$h = \frac{l}{N}$$

$$k = \frac{10.6}{1.50} - 1 = 6.07$$

$$N = 5.54 \left( \frac{10.6}{1.45} \right)^2 = 296$$

$$h = \frac{1000}{296} = 3.38 \text{ cm}$$

Finally, calculate the resolution between the benzene and toluene peaks.

$$R_s = 1.18 \frac{(t_{toluene} - t_{benzene})}{(W_{1/2 toluene} + W_{1/2 benzene})}$$

$$= 1.18 \frac{(10.6 - 7.45)}{(1.05 + 1.45)}$$

$$= 1.48 \text{ (quantitative separation)}$$

Plate theory assumes that a column is mathematically equivalent to a plate column.

An equilibrium is established for the solute between the mobile and stationary phases one each plate.

It is a useful theory and can predict many aspects of chromatographic performance.
A partial differential equation set up by van Deemter for a linear isotherm resulted in an effluent concentration function.

It is based on a Gaussian distribution similar to that of plate theory.

He was attempting to account for the dynamics of the separation process.

The equation consists of three basic terms.

\[ H = 2 \lambda \, d_p + \frac{2 \gamma D_g}{u} + 8 \frac{k d_i^2}{\pi^2 (1+k)^2 D_l \, u} \]

- \( \lambda \) - factor characteristic of packing
- \( d_p \) - particle diameter
- \( \gamma \) - factor for irregularity of interparticle spaces
- \( D_g \) - diffusion coefficient of compound in gas
- \( D_l \) - diffusion coefficient of compound in liquid
- \( k \) - capacity ratio
- \( d_i \) - liquid phase effective film thickness
- \( u \) - linear gas velocity
- \( H \) - height of a theoretical plate

We commonly group the various constants into single terms and reduce the equation to:

\[ H = A + \frac{B}{u} + Cu \]

- \( A \) - multipath or eddy diffusion
- \( B \) - molecular diffusion
- \( C \) - resistance to mass transfer

Note that \( A, B \) and \( C \) are constants but the effect of \( B \) and \( C \) is dependent of the velocity of the mobile phase.

The goal is to find \( H_{\text{min}} \) for optimum column performance.

\[ H = A + \frac{B}{u} + Cu \]

For a packed GC column
You don’t need to calculate H for each column/eluent combination to be able to use this relationship.

An understanding of the effects of each term will help you design/select appropriate columns and optimum flows.

**A term**

**Multipath or eddy diffusion**
This term accounts for the effects of packing size and geometry.

The range of possible solute paths results in a minimum peak width.

**B term**

**Molecular diffusion**
Represents broadening due to diffusion in the mobile phase.

Reverse diffusion is more significant than forward to mobile phase movement.

**C term**

**Resistance to mass transfer.**
It takes time for a solute to reach an equilibrium between the mobile and stationary phases.

Thick or viscous stationary phases have larger C terms.
You can minimize the effect of the C term by:

- Using “thin” coatings of the stationary phase on a solid support.
- Use less viscous phases.
- Keep the flow as low as possible - limited by the effect of the B term.

The best velocity is a function of the Van deemter equation and practical conditions. You need to have a useable analysis time.

Also, since the effects of B are greater than C, it is best to set the flow a little on the high side in case it changes slightly during the analysis.

This plot is for a typical packed column GC analysis. Gases have high diffusions and column packings are relatively large.

Not much effect from A or C. There is no packing and the phase is very thin.

At first, LC relied on irregular packing. Now the packings are pretty good so the A term is very low.

The B and C terms are low because liquids diffuse much more slowly than gases.

We typically think of GC and LC as quantitative tools.

In general, chromatography is a 'blind' method. It indicates the presence of a substance but not what it is.

Qualitative data can also be obtained even with non-discriminating detectors.
Retention data - can be used for some qualitative work. The $t_R$ is characteristic of a substance, compared to a standard. To be useful, some problems must be addressed.

- Reproducibility of absolute retention data depends on several experimental conditions.
- Is $t_R$, $v_R$, $v'_R$ or $t'_R$ best to use?

**Retention volumes**

If the flowrate ($F_c$) is constant and known then:

- **Retention volume** = $V_R = t_R F_c$
- **Adjusted $V_R$** = $V'_R = t'_R F_c$
- **Hold up volume** = $V_m = t_m F_c$

**Retention relationships**

Retain volume or time may be used for identification.

For a homologous series, $V_R$ can be accurately determined by:

$$\ln V'_n = a + bn$$

where $V'_n$ = adjusted retention volume
- $n$ = carbon number
- $a, b$ = fit parameters

and $V'_n = V_n - V_m$

**Absolute retention index**

For non-paraffins, we can simply calculate an index value like it was a paraffin. The value does not need to be a whole number.

$$I_p = n_1 + \frac{\ln V_x - \ln V_{n1}}{\ln V_{n2} - \ln V_{n1}}$$

$n_2 > x > n_1$

$n_2$ and $n_1$ are reference paraffins.
Kovat's retention index

A modification of the absolute index where:

$$I_K = 100 \cdot I_P = 100n_1 + 100 \cdot \frac{\ln V_x - \ln V_{n1}}{\ln V_{n2} - \ln V_{n1}}$$

This index has been determined at different temperatures for a large number of compounds. Tables are also available.

The value can be used to compare related separations.

Relative retention data

One practical approach for your own data is the use of relative retention.

$$r_{i,\text{std}} = \frac{t'_{R(u)}}{t'_{R(\text{std})}} = \frac{V'_{R(u)}}{V'_{R(\text{std})}} = \frac{k(u)}{k(\text{std})}$$

This is the most common approach. It only requires a single standard.

Relative retention data

To be useful

Standard should be a part of the sample or added to it - *internal standard*

It should be something that elutes near center of an analysis - although you can use more than one.

Sample size should be small.

Values will remain pretty constant between runs - may vary wildly with a new column.

Retention time, $t_R$

Simple retention time data is adequate for simple assays like process quality control.

- You already know what is there.
- There are only a few components in the sample (or only a few of interest).

If a true unknown is observed, you can’t do much more than note its presence!

Quantitative analysis

All chromatographic detectors produce a signal that drives a meter, recorder, integrator or A/D converter.

While the detectors used for GC and LC are not the same, quantitation methods are identical.

Each detector will produce a response/unit concentration. This is substance dependent so **standards must always be used.**

Peaks

Each method of quantitation assumes that you have one or more reasonably resolved peaks.

You must be able to find the beginning and end of each peak as well as it’s maximum.
In some cases, you can assume that peak height is proportional to concentration.

**Advantages**
- Simplicity
- Rapid calculations

**Disadvantages**
- Height is more variable than area
- Typically used only with capillary columns

This is the major approach for establishing a relationship between peaks and concentration.

\[ \text{Area} \propto \text{Concentration} \]

Area is determined from a large number of measurements and detectors usually have very large dynamic ranges. This results in a very reliable measurement.

A device used to trace the peak. It produces a number that is proportional to peak area.

**Major problem**
If the peak is approximately Gaussian, how do we accurately measure its area?

**Manual**
- Cut & Weigh
- Planimeter
- Triangulation

**Automated**
- Integrating recorder
- Digital integrators
- Computer systems

With this approach, each peak is cut from the recording paper and weighted. Weight is then considered proportional to area.

**Main manual method.**
Assumes that each peak approximates a triangle. Area can be determined by

\[ \text{area} = \text{peak height} \times \text{width} \]

or

\[ \text{area} = \text{peak height} \times 2 \times \text{width}_{1/2} \]
**Integrating recorders**

A special two pen recorder.
- The first pen tracks the chromatographic signal.
- The second traces a series of zigzags.

The larger the peak response gets, the more rapidly the second pen sweeps back and forth.

The total number of zigs and zags can then be related to the peak area.

If the peak gets too large, the second pen stops moving. The peak must be kept within the working range.

**Digital integrators**

Rely on A/D conversion of detector response.

**Caution!**

Peaks are typically processed for area on the fly.

This includes not only peak detection but may also handle other methods for dealing with poorly shaped or resolved components.

If a peak is missed, the run must be repeated.

**Computer systems**

Include the same methods of peak detection and integration as integrators.

Major advantage is that the entire chromatographic run is stored prior to analysis.

This allows you to test out methods of integration on a single run and to reanalyze data if a peak is missed.

**Summary**

<table>
<thead>
<tr>
<th>Method</th>
<th>Time, min</th>
<th>Precision, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planimeter</td>
<td>15</td>
<td>4.1</td>
</tr>
<tr>
<td>Triangulation</td>
<td>10</td>
<td>2.5 - 4</td>
</tr>
<tr>
<td>Cut &amp; weigh</td>
<td>20</td>
<td>1.7</td>
</tr>
<tr>
<td>Int. Recorder</td>
<td>5</td>
<td>1.3</td>
</tr>
<tr>
<td>Integrator</td>
<td>N/A</td>
<td>0.44</td>
</tr>
<tr>
<td>Computer</td>
<td>N/A</td>
<td>0.44</td>
</tr>
</tbody>
</table>
Quantitative interpretation

OK, now you have all of your peak areas.
Let's assume you knew what you were doing and all the areas were measured properly.

Big deal!
A relationship between concentration and area must be established or we're just spinning our wheels.

Determining concentration

Several approaches can be used. Use the one that is most appropriate for your method.

Methods we'll cover
External standard method
Internal standard method

External standard method

Requirements for proper use:
Standard solution containing all eluents to be quantified.
Standard eluents should be of similar concentration as unknowns.
The standard and sample matrix should be as similar as possible
Analysis conditions must be identical - stable instrument, same sample size ...

Example - determination of X in MeCl₂
Prepare a standard of X
(20.0 mg in 100 ml MeCl₂) - 0.200 µg/µl
Use an injection volume of 5 µl for both the standard and the unknown.
Measure the areas produced by both the sample and the unknown.
Area X_{std} = 2000 units
Area X_{unk} = 3830 units

Now, determine the concentration of X in you unknown.

\[ \text{conc}_{\text{unknown}} = \frac{\text{Area}_{\text{unknown}}}{\text{Area}_{\text{known}}} \times \text{conc}_{\text{known}} \]

conc_{unknown} = \frac{3830}{2000} \times 0.200 \, \mu g / \mu l
= 0.384 \, \mu g / \mu l

You can now convert to a more appropriate concentration if required.
Regardless of the method for introducing the standard or calibrating, the calculations are the same. They are the same as with the detector response factor method.

Our NORM substance is now predetermined and a fix value.

\[ C_{\text{STD}} = f_{\text{STD}} A_{\text{STD}} \]

\[ C_{\text{unk}} = f_{\text{unk}} A_{\text{unk}} \]

Since the internal standard is assigned a value of 1.00 and is held constant, we can correct for run to run variations by:

\[ C_{\text{unk}} = \frac{A_{\text{STD1}}}{A_{\text{STD2}}} \frac{A_{\text{unk}}}{A_{\text{known}}} C_{\text{known}} \]

known & ISTD1 are obtain from the standard, unk & ISTD2 from the unknown.

Accounts for factors such as:

- Sample injection errors or changes
- Slow detector variations
- Slow column changes
Now, inject your unknown.  
\[ \text{Area}_X = 990 \]
\[ \text{Area}_{\text{ISTD}} = 1031 \]
\[ C_X = \left( \frac{1009}{1031} \right) \left( \frac{990}{635} \right) \times 11.3 \text{ mg} \]
\[ = 17.24 \text{ mg} \text{ X in the unknown.} \]