

حل 01

(1) طويلة z_1 و عمدة للعدد المركب z_1 :

$$|z_1| = \sqrt{1^2 + (\sqrt{3})^2} = 2 \bullet$$

$$\begin{cases} |z_1| = 2 \\ \text{Arg}(z_1) \equiv \frac{\pi}{3} [2\pi] \end{cases} \text{ ومنه } \begin{cases} \frac{1}{2} = \cos \frac{\pi}{3} \\ \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \end{cases} \text{ إذن } z_1 = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) \text{ منه}$$

طويلة z_2 و عمدة للعدد المركب z_2 :

$$|z_2| = \sqrt{1^2 + 1^2} = \sqrt{2} \bullet$$

$$\begin{cases} |z_2| = \sqrt{2} \\ \text{Arg}(z_2) \equiv \frac{\pi}{4} [2\pi] \end{cases} \text{ ومنه } \begin{cases} \frac{\sqrt{2}}{2} = \cos \frac{\pi}{4} \\ \frac{\sqrt{2}}{2} = \sin \frac{\pi}{4} \end{cases} \text{ إذن } z_2 = \sqrt{2} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) \text{ منه}$$

(2) طويلة z_1 و عمدة للعدد المركب z_0 :

$$\text{Arg} \left(\frac{z_1}{z_2} \right) \equiv \text{Arg}(z_1) - \text{Arg}(z_2) \equiv \frac{\pi}{3} - \frac{\pi}{4} \equiv \frac{\pi}{12} [2\pi] \text{ و } |z_0| = \frac{|z_1|}{|z_2|} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ إذن } z_0 = \frac{z_1}{z_2}$$

$$\begin{cases} |z_0| = \sqrt{2} \\ \text{Arg}(z_0) \equiv \frac{\pi}{12} [2\pi] \end{cases} \text{ إذن}$$

(3) كتابة z_0 على الشكل الجبري

$$z_0 = \frac{1+i\sqrt{3}}{1+i} = \frac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)} = \frac{\sqrt{3}+1}{2} + i \frac{\sqrt{3}-1}{2}$$

(4) استنتاج $\cos \frac{\pi}{12}$ و $\sin \frac{\pi}{12}$

$$z_0 = \sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \text{ هو الشكل المثلثي لـ } z_0$$

$$z_0 = \frac{\sqrt{3}+1}{2} + i \frac{\sqrt{3}-1}{2} \text{ هو الشكل الجبري لـ } z_0$$

$$\begin{cases} \cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4} \\ \sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4} \end{cases} \text{ أي } \begin{cases} \sqrt{2} \cos \frac{\pi}{12} = \frac{\sqrt{3}+1}{2} \\ \sqrt{2} \sin \frac{\pi}{12} = \frac{\sqrt{3}-1}{2} \end{cases} \text{ نستنتج}$$