حلّ -03-

(1 $\left|\sqrt{3}+i\right| = \sqrt{\left(\sqrt{3}\right)^2+1^2} = 2$ إذن $\sqrt{3}+i = 2\left(\frac{\sqrt{3}}{2}+i\,\frac{1}{2}\right) = 2\left(\cos\frac{\pi}{6}+i\,\sin\frac{\pi}{2}\right) = 2e^{i\frac{\pi}{6}}$

$\left|\sqrt{2}+i\sqrt{2}\right| = \sqrt{\left(\sqrt{2}\right)^2+\left(\sqrt{2}\right)^2} = 2$ إذن $\sqrt{2}+i\sqrt{2} = 2\left(\frac{\sqrt{2}}{2}+i\,\frac{\sqrt{2}}{2}\right) = 2\left(\cos\frac{\pi}{4}+i\,\sin\frac{\pi}{4}\right) = 2e^{i\frac{\pi}{4}}$

منه $Z = \frac{2e^{i\frac{\pi}{6}}}{2e^{i\frac{\pi}{4}}} = e^{i\frac{\pi}{6}-i\frac{\pi}{4}} = e^{-i\frac{\pi}{12}}$.

2) نضع $z = r\,e^{i\alpha}$.

المعادلة $z^2 = \frac{\sqrt{3}+i}{\sqrt{2}+i\sqrt{2}}$ تكافئ $r^2e^{i\,2\alpha} = e^{-i\frac{\pi}{12}}$ أي $r^2e^{i\,2\alpha} = e^{-i\frac{\pi}{12}}$

نستنتج $\begin{cases} r^2 = 1 \\ 2\alpha = -\frac{\pi}{12}+2k\pi \end{cases}$ و $k \in \mathbb{Z}$ إذن $\begin{cases} r = 1 \\ \alpha = -\frac{\pi}{24}+k\pi \end{cases}$

من أجل $k = 0$: $\alpha = -\frac{\pi}{24}$ منه $z = 1\times e^{-i\frac{\pi}{24}}$

من أجل $k = 1$: $\alpha = -\frac{\pi}{24}+\pi = \frac{23\pi}{24}$ منه $z = 1\times e^{i\frac{23\pi}{24}}$

تقبل المعادلة $z^2 = \frac{\sqrt{3}+i}{\sqrt{2}+i\sqrt{2}}$ حلين هما $e^{-i\frac{\pi}{24}}$ و $e^{i\frac{23\pi}{24}}$.